

# Suppliers Searching and Market Concentration

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It is a very preliminary draft, so there can be notes and results that are incomplete.

## **Abstract**

I analyze how falling search costs - driven by improvements in communications and transportation technology since the late 1990s - have affected firm size distribution and allocative efficiency. Lower costs have enabled smaller firms to source inputs internationally, reflected in the declining size of median and lower-percentile importers. Meanwhile, top importers have also grown, driving up market concentration. I suspect these trends originate from the structure of the intermediate goods market. In Swedish import data, I find that input prices exhibit rising variance and are negatively correlated with productivity. Based on my empirical findings, I develop a quantitative search-and-bargaining model in which search costs serve as both expansion cost and entry barrier. I find that the decrease of search cost sometimes favor the most productive firms and sometimes smaller ones. Consequently, market concentration and efficiency respond non-monotonically to changes in search costs.

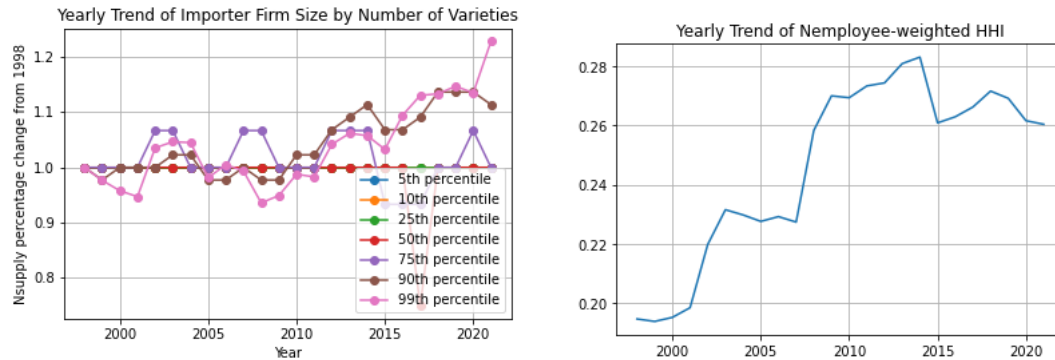
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# 1 Introduction

Significant advances in communication and transportation technologies have made it easier for even smaller firms to expand their supplier networks internationally. Nevertheless, concentration in the Swedish manufacturing sector has risen. This finding is consistent with extensive existing literature, such as Autor et al. (2020). In other words, although smaller firms likely gain from falling search costs, larger firms continue to outgrow their smaller counterparts in sales.

Researchers have explored various explanations for this phenomenon, from increasing entry barrier (Covarrubias et al. (2020)), import competition (Amiti and Heise (2025)) or falling overhead cost resulting from information technology advancements (Aghion et al. (2023)). In this paper, I argue that supply-chain market structure and associated search costs also play a crucial role. The graph below illustrates the number of varieties (product  $\times$  country) imported by each Swedish manufacturing firm. While the median and lower-percentile firms have experienced virtually no change in the number of imported varieties from 1998 to 2021, the largest firms have increased their imported varieties by approximately 10  $\sim$  20%. I propose that decreasing search costs significantly influence market concentration.



The contributions of this paper to existing literature are threefold:

Firstly, this work contributes to the literature on increasing market concentration, complementing studies by Autor et al. (2020) and Covarrubias et al. (2020), which primarily focus on the United States. By examining the Swedish manufacturing sector, I provide additional empirical evidence. While prior literature identifies channels such as import competition (Amiti and Heise (2025)) and decreasing overhead costs (Aghion et al. (2023)), I emphasize the crucial role of reduced search costs within input markets.

Secondly, this paper relates closely to the literature examining input price dispersion. Substantial dispersion in input prices has recently been documented across various contexts and countries, notably by Atalay (2014). I observe the same dispersion in Swedish import data and find that price dispersion has been increasing in recent years. The cause of such price dispersion can be buyer market power as in Morlacco

(2020) or Rubens (2023). It can also be quality difference as proposed by Kugler and Verhoogen (2012), the fitness of match as found out by Burstein et al. (2024). I summarize some of the aforementioned channels that drives firms input price, document correlations between input price gap and firms' characteristic and collect other relevant empirical observations. I suspect that one possible potential explanation for the increasing price dispersion is the decreasing trade cost.

I also contribute to misallocation literature pioneered by Restuccia and Rogerson (2008) and Hsieh and Klenow (2009). Specifically, I explore how supply-chain search costs affect allocative efficiency for downstream firms. I construct a theoretical model featuring search-and-bargaining between buyers and supplier which generates endogenous price gaps and analyze the effect of this dispersion on aggregate output. This provide new insights as the wedge now depends on firms' productivity.

## 2 Data

I mainly use Swedish administrative data provided by the Statistic Sweden. The main data block is the import part of the *Utrikeshandel med varor*, Foreign Trade in Goods dataset. It includes import records on the firm-year-8digit\_product-country level during the period 1998-2021. For example, if the company import german cars in 2018 from two different suppliers, I can only see one aggregated record.

It includes all imports records during the period 1997-2018 for imports from outside of EU (Switzerland or EEA countries included) and the intra-EU importing records of companies which import at least SEK 9.0 million worth of goods(See SCB (2018) for more). Each record entry includes the importer ID, origin country of the purchase, value, weight and the 8 digit level product code of the imported product. The customs data also includes, for a subset of products, an additional variable "Other Quantities" (e.g. pieces for pencils or  $m^2$  for curtains in 2024 etc.)<sup>1</sup>. I also use firm level balance sheet data to obtain importers' characteristics, such as sales, wage bill and productivity etc.

Each entry includes value and quantity of the imported product. I define the product-country pair  $pc$  as an variety. And I define price as the log residualized price

$$P = \log\left(\frac{P_{fpcy}}{P_{pcy}}\right)$$

where the numerator is the deflated value over quantity of one data entry and the denominator is the within-variety-year weighted-average price

$$\bar{P}_{pcy} = \frac{\sum_{pcy} \text{Deflated Value}}{\sum_{pcy} \text{Quantity}}$$

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<sup>1</sup>See the subset of products in the Övrigt om varukoder section on <https://www.scb.se/lamna-uppgifter/undersokningar/intrastat-in--och-utforsel-av-varor/varukoder-kn-for-uppgiftslamnande-till-intrastat/>

I deflate the value using aggregate CPI data to adjust for inflation.

For some analysis, I construct a firm-level price index, which is the weighted average across all variety the firm buy within a year:

$$\frac{\sum \text{Price} * \text{Deflated Value}}{\sum_f \text{Deflated Value}}$$

For example, if a company buy 10000SEK worth of german cars at 20% higher unit price than average german cars buyer and 3000SEK worth of chinese headphones at 30% lower price than average. Then, their firm level price index is:

$$\frac{1.2 * 10000 + .7 * 3000}{10000 + 3000}$$

## 2.1 Summary Statistics (WIP)

I did basic data cleaning by excluding observations where the total value is below 100 SEK (around 9USD). I suspect those records are either not arm-length trade, a typo or just a placeholder value.

To show that imported goods are important for Swedish economy, I use the TiVa dataset provided by OECD and calculate 2 statistics - percentage of Foreign Value Added in Swedish End-Consumption and percentage of Foreign Value Added in Swedish Gross Export, and the percentage of intermediates in import. Both Values are non-trivial:

Trade in Value Added (TiVA) 2023 edition, year 2020

domestic value added in Swedish manufactured product: 69% (75.5% DVAR in all sectors, OECD)

Foreign Value Added in Domestic Final Demand in All Sector (Manufacturing): .26 (.52)

intermedaites in Import, All Sector (Manufacturing): .38 (.36)

## 3 Empirical Observations

From the data, I document 4 main empirical facts:

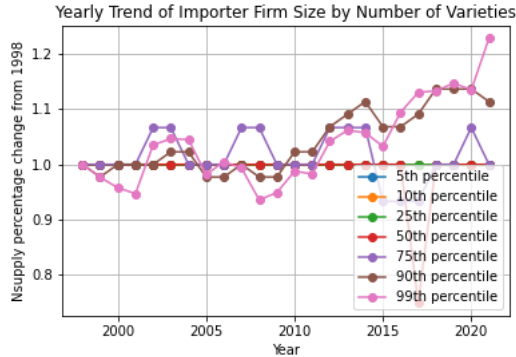
- Variety increase for biggest firms, but not for median firms

- Importer firm size has been dispersing. Median importing firm has become smaller over time. This regularity are suggestive evidences that trade cost has been decreasing.
- Input price dispersion is substantial and has been on the rise. The variance of log residualized price has been increased from around 1.3 in 2000 to around 1.7 in 2020. Also, across-firm component can only explain  $\sim 15\%$  of the dispersion in each year when I perform variance decomposition on the regression  $P \sim \alpha + FE_{firm}$ . Of course, it is still likely that a non-negligible part of the dispersion is driven by mechanisms that is not covered in this paper, such as business cycle or exchange rate fluctuations. I am currently investigating further into it.
- Productive firms receive a lower weighted-average input price, but not a lower input price in every variety. Also, productive firms import more varieties on average.

### 3.1 Trend of Variety Imported

TODO: BroadBand Rollout Event Study a la Malgouyres et al. (2021)

I define a "variety" as a unique country-product pair and measure "firm size" based on annual firm sales. In the graph presented, I compare the number of varieties imported each year across firms of different sizes. To facilitate comparison across firm-size percentiles, I normalize the annual number of varieties by their respective values from 1998. I observe that, except for the largest firms, most firm-size percentiles have not experienced significant changes in the number of imported varieties. It remains unclear whether this pattern is driven by the entry of smaller firms. **TODO:** Graph that shows only incumbents (balanced panel). In contrast, the largest firms (specifically, the 90th and 99th percentiles) have increased the variety of imports by up to 20%. Given that the number of varieties imported directly relates to search activities, this observation suggests differential effects of search costs across firm sizes.



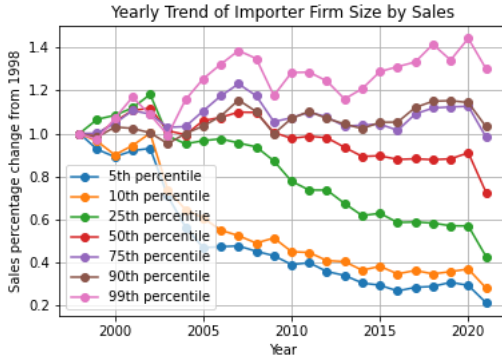
### 3.2 Importing Firm Size Dispersion

In this section, I examine changes in firm size, measured by annual sales, at various percentiles over the period from 1998 to 2021. Specifically, I analyze how firm sizes at different percentiles evolve relative to their 1998 levels by normalized by their respective values from 1998.

The graph clearly shows a significant reduction in firm size at the median and lower percentiles. For instance, the 5th percentile firm in 2021 is approximately 80% smaller than the 5th percentile firm in 1998. This substantial decrease likely results from reduced search costs, which acted as a barrier to entry into the import market. Lower search costs may have allowed less productive firms, which previously faced prohibitively high barriers, to enter the market. It is an indicator that entry cost to the international supply chain has reduced drastically. **TODO:** Graph that shows only incumbents (balanced panel).

In contrast, firm sizes at higher percentiles have substantially increased, by up to 40% at the 99th percentile, demonstrating the rapid growth potential of the largest firms. This growth can likely be attributed to reduced search costs facilitating expansion through the addition of new suppliers for incumbent, larger firms, which is explained in the previous section.

Overall, these findings indicate increasing dispersion in firm sizes within the market. A potential contributing factor can be falling search costs, which simultaneously influence entry barriers and expansion costs, thereby affecting both the intensive and extensive margins of the firm-size distribution.

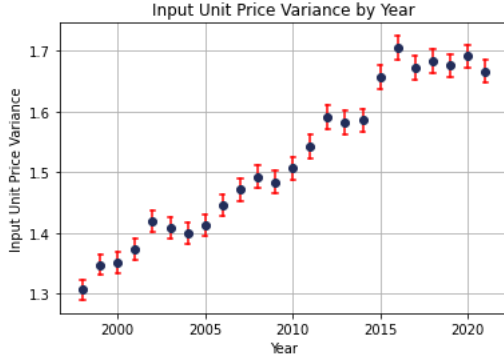


### 3.3 Decreasing Search Cost for Suppliers and Input Price Dispersion

Related to the decreasing search cost and the influx of smaller firms into the import market is the input price dispersion. As the distribution of importer size become more disperse, variance of input price is also increasing. I define input price dispersion as:

$$\text{Var}(P) = \text{Var}\left(\log\left(\frac{P_{fpcy}}{P_{pcy}}\right)\right)$$

I drop some of the outliers, by setting a  $P$  cutoff at XXX. The increasing trend and its magnitude is robust to such cleaning. It is also robust when I limit the analysis to within-EU import or imports from most of Sweden's major trading partner.



One potential link between the two dispersing distribution is that input unit price is inversely correlated to firm size. When there are a lot of small firms goes into the import market, they buy very little and they are not familiar with the market, therefore they will pay a relatively high price. On the other hand, big firms become bigger, gain market power and face a even lower price. In this case, the price gap will grow. I will present more evidence below.

It is also important to note that quality differences is likely not the reason. I measure the price variance driven by quality difference by comparing price variance within differentiated products and non-differentiated ones, as defined by Rauch (1999). It is a very conservative measure that likely exxagerate the important because this gap likely include other channel such as bargaining and information frictions. But what is important is that the quality gap hasn't been increasing during the period, neither does the composition of differentiated products in all products. See more in Appendix.

### 3.4 Buyers' Productivity and Input Price

In this section, I primarily examine how a firm's output productivity influences the prices it face in the input market. To understand this, I have to first define a measure for productivity. I define productivity as the quality adjusted physical productivity TFPQ. First, I use balance sheet data to estimate revenue-based productivity TFPR under Cobb-Douglas assumption:

$$TFPR_f = \frac{(p_f y_f)}{(w_f l_f)^{\alpha_l} (r K_f)^{\alpha_k} (t_f X_f)^{1-\alpha_l-\alpha_k}}$$

where  $py$  is sales,  $wl$  is the wage bill,  $K$  is capital and  $tX$  is intermediate cost, which are all observables on the balance sheet. I assume the capital rental rate  $r = 0.15$ ,

which is standard estimates from the literature. The remaining unknown parameters  $\alpha$ s can be easily backout using the Cobb-Douglas property where  $\alpha$  equal to the expenditure share of that input under cost minimization. I assume  $\alpha$  is constant within industry. I then estimate the quality-adjusted TFPQ by:

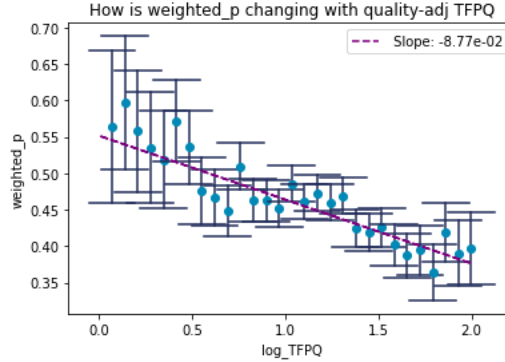
$$\text{TFPQ}_f = \frac{p_f}{Q_f} \text{TFPR}_f$$

, where  $p$  is the price and  $Q$  is the quality of the firm's production.  $p$  can be obtained from the producer price index (PPI) or export database. One caveat is that the PPI database covers only the largest firms, and larger companies are typically those that tend to export. Consequently, the sample analyzed in this section is biased toward larger firms. Unfortunately, quality  $Q$  is generally not observable. To obtain an estimate of  $Q$ , I rely on the theory. Assuming the monopolistic competition in final good demand, the household optimization gives:

$$\frac{p_i y_i}{p_j y_j} = \left( \frac{p_i / Q_i}{p_j / Q_j} \right)^{1-\epsilon}$$

where Sales  $py$  and price  $p$  are observables in the data. I can then back out  $Q$  by applying the standard elasticity  $\epsilon = 5$ .

Now that I get an estimate for TFPQ, I can start my analysis. The firm level price index is negatively correlated to TFPQ. That means more productive firms pay lower unit price compared to their less productive competitors for the same input bundle.



It is interesting because this correlation means that on top of their productivity advantage, more productive gain further step ahead to their less productive counterpart in the input market. This correlation widened the gap between productive and unproductive firms. This potentially make the variance of firm size distribution larger and can have implication of increasing concentration.

Note that this relationship doesn't hold when we replace TFPQ by sales or TFPR. The reason is simple. Sales or TFPR is positively correlated with the output price  $p$  and output price  $p$  is positively correlated to marginal cost and input cost, which



is the independent variable. Therefore, the negative correlation we see in the figure above maybe cancelled out/dampened or even dominated by this positive correlation. However, this result is robust when TFPQ is not quality-adjusted (See Appendix).

### 3.5 Short Summary

On top of the 4 main empirical facts, I document also:

- Firms that receive a lower weighted-average input price tends to have higher profit share.
- More productive firms import more varieties
- Price and quantity are inversely correlated

These data moments are used in model calibration and are further explained in appendix.

In conclusion, I have look into the data to uncover some important charactersitic that intermediate input market exhibit. First of all, there is an inverse relationship between productivity and weighted average unit price, but no relationship between productivity and just unit price. Also, for realized transactions, price and quantity are inversely correlated. There are also big within firm price difference and large quality differences. Among all these, search costs seem to play an important role.

On top of my own findings, I also adopt 2 additional features in my model from recent trade literature. The first one concerns the time dependency of the supplier network. Martin et al. (2023) suggests that an median buyer-supplier relationship last around an year. Therefore, as my model period is an year, having a repeated static model is not unreasonable and simplifies the numerical exercises by a lot. The second is related to inventory. Alessandria et al. (2010) point out that average company import internationally every 150 days, which indicate that inventory also should not matter for most firms in a yearly model.

## 4 Quantitative Model

Based on my empirical findings, I build a intermediate good market that features search and bargaining on top of a standard monopolistic competitive model.

The model timing is as followings: All productivities (and quality for seller) realize and are observed by the buyer  $d$  and seller  $u$  themselves according to some commonly known distribution. Buyers start from intermediate good  $XQ = 0$ , where  $Q$  is the corresponding quality. Given the intermediate good contracted  $XQ$  and self-productivity,

buyers pay search cost  $\kappa(n)$  or drop out from searching. The searching buyers will randomly meet a seller. Sellers produce with linear technology:

$$x_u q_u = z_u l$$

with labor as its only input. Productivity  $z_u$  and quality  $q$  are observable to the buyer. The matched firms will (Nash) bargain over a contract which specify intermediate good quantity  $xq$  (can be nothing) and total price  $T$ . After a contract is signed, buyer can stop or pay convex search cost  $\kappa(n+1)$ , which has the functional form:

$$\kappa(n) = \kappa_0(1 + \tau)^n$$

After all firms stop searching, the buyer will be endowed with intermediate good  $Q_d X_d = (\int q_u^\rho x_u^\rho du)^{\frac{1}{\rho}}$ , where  $Q_d$  is the quality. Buyers produce consumption good with Cobb-Douglas technology:

$$Q_d^\alpha y_d = z_d (Q_d X_d)^\alpha l^{1-\alpha}$$

by choosing labor and household consumes final good  $Y = (\int (Q_d^\alpha y_d)^{\frac{\epsilon}{\epsilon-1}} dd)^{\frac{\epsilon-1}{\epsilon}}$ . When next period starts,  $X$  is reset. Search cost  $\kappa_t$  is decreasing every period.

The model can be summarized into 4 value functions, which represent search decision:

$$V(z, XQ, n) = \max\{V^s(z, XQ, n), V^{NS}(z, XQ)\}$$

The buyer with productivity  $z$  with contracted inputs  $XQ$  have to decide if she want to have her  $n$ th search by comparing value function  $V^s$  and  $V^{NS}$  for searching and not searching respectively. She will choose the decision that brings her bigger expected value.

The search value is:

$$V^s(z, XQ, n) = \int V^m(z, z_u, XQ) dF(z_u) - w\kappa_t(n)$$

where  $\lambda > 0$ .

The not-search value is:

$$\begin{aligned} V^{NS}(z, XQ) &= \pi(z, XQ) \\ &= p^* y^* - w \left( \frac{y^*}{z(XQ)^\alpha} \right)^{\frac{1}{1-\alpha}} \\ &= \left[ \frac{1 + \alpha(\epsilon - 1)}{\epsilon} \right] \left( \frac{w\epsilon}{(\epsilon - 1)(1 - \alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P-\epsilon} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha Q^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \end{aligned}$$

The not-searching value is simply the profit that can be obtained based on the exiting input  $XQ$ . Given  $XQ$  and the demand system, the firm solve the deterministic profit maximzing function:

$$\begin{aligned} \max_{p,y} & py - w\left(\frac{y}{zX^\alpha Q^\alpha}\right)^{\frac{1}{1-\alpha}} \\ s.t. & yQ^{\alpha(1-\epsilon)} = C\left(\frac{p}{P}\right)^{-\epsilon} \end{aligned}$$

The output price, output quantity, labor usage and thus the profit function all have closed form solutions.

The matching value is:

$$\begin{aligned} V^m(z, z_u, XQ) &= -T(z, z_u, XQ) + V(z, X^N Q^N) \\ X^N &= (X^\rho + x^\rho)^{\frac{1}{\rho}} \\ Q^N &= \frac{1}{X^N} \left( \sum_{u \in U_i} x_u^\rho q_u^\rho \right)^{\frac{1}{\rho}} \end{aligned}$$

The bargaining problem:

$$\max_{T, xq} (-T + V(z, X^N Q^N) - V(z, XQ))^\theta \left(T - w \frac{xq}{z_u}\right)^{1-\theta}$$

In this Nash bargaining framework, the buyer and supplier negotiate over the payment  $T$  (from the buyer to the supplier) and the quality-adjusted input quantity  $xq$ . The buyer possesses a bargaining power of  $\theta$ , and its surplus is defined as the value gained from a successful negotiation minus the payment. The seller's surplus, on the other hand, is the payment received minus the production costs required to fulfill the contract. Due to the linear nature of the production technology, the bargaining process does not affect the seller's outside option, which can therefore be set to zero.

In the general equilibrium model, good and labor markets should be clear and budget constraint for the household should be satisfied.

The problem for the 2 types of household :

$$\begin{aligned} \max_{c_i} & C = \left( \int c_i^\epsilon di \right)^{\frac{1}{\epsilon}} \\ s.t. & PC \leq wL_t && \text{factor owner} \\ & PC \leq \pi && \text{firm owner} \end{aligned}$$

Factor Market Clearing:

$$\int \mathbb{E}[l + \kappa|z]dz = \bar{L}_t$$

Good Market Clearing:

$$\int \mathbb{E}[y|z]dz = C$$

I solve the model by a standard VFI procedure. In the PE model, I first make an initial guess on the search value function  $V^S$  and calculate not-search  $V^{NS}$  based on the closed form solution above. Based on  $V^S$  and  $V^{NS}$ , I solve the firm search policy and get a guess  $V$ . Assuming this guess of  $V$  is true, I solve the policy functions for payment  $T$  and quality-adjusted quantity  $xq$  by grid searching. Then, I calculate the matching value  $V^m$  by plugging in the optimal  $T$  and new state  $X^N Q^N$  and update the expected search value  $V^S$  and  $V$ . The process stops when the value functions converge.

To speed up the grid searching, I make a proof that the bargaining problem is strictly concave in  $T$  (See Appendix). In this way, I reduce the 2D optimization to 1D.

## 5 Results

### 5.1 Restrictive Model

I start with looking at an restrictive model where firms can only have one supplier (variety), the search cost  $\kappa$  is constant instead of convex, no quality differentiation and both upstream and downstream productivity follows a simple  $U(0, 1)$  distribution. It become a quite familiar model of McCall (1970) random job search model. With this class of model, we can solve the optimal stopping problem by getting the reservation productivity  $z_u^R(z)$  and  $z^R$ . I can get an idea how search cost  $\kappa$  affect firm size distribution from a fixed point solution.

The reservation supplier productivity is solved by 2 equations. The first equation is the definition of the outside option  $D$ :

$$D = \int_{z_u^R}^{z_u} [\pi(z_u) - T(z_u)]dF(z_u) + \int_{z_u}^{z_u^R} DdF(z_u) - w\kappa$$

which consists of 3 terms: The first one is expected profit given the probability of meeting a satisfactory supplier ( $z_u \geq z_u^R$ ), the second one is falling back to outside

option if the supplier is worse than the reservation productivity and the last term is minus the search cost.

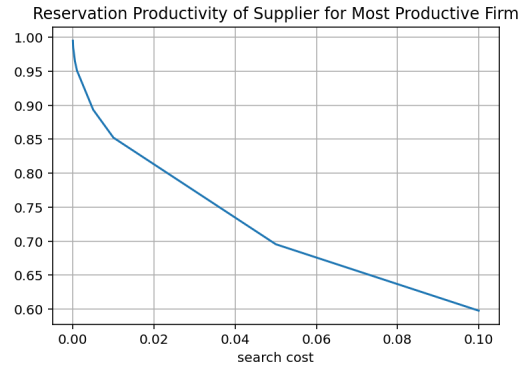
The second equation is defined by the indifference condition, where for any buyer, she should be indifferent between the profit for meeting the reserved supplier and going into outside option:

$$\pi(z, z_u^R) - T(z, z_u^R) = D$$

Combining the equations we get a expression for solving  $z_u^R$  for each  $z$ :

$$\frac{\kappa}{Kz^{\epsilon-1}} = [1 + \alpha(\epsilon - 1)z_u^{R[\alpha(\epsilon-1)+1]} - (\alpha(\epsilon - 1) + 1)z_u^{R\alpha(\epsilon-1)}]$$

This expression provides a relationship between  $z_u^R$  and  $\kappa$  for any firm with productivity  $z$ . And solving this fixed point formula gives:

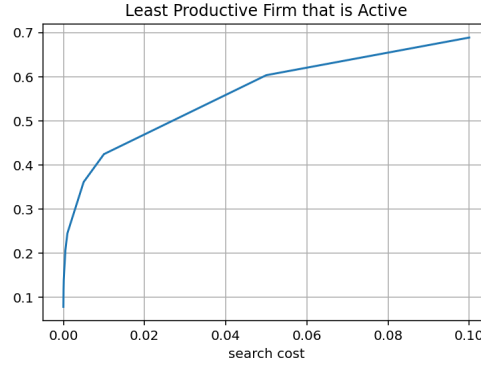


This shows that when search cost decrease, the reservation productivity, given  $z$ , is increasing. In other words, when search cost decrease, the expected productivity of the supplier increase. It is obvious that matching with a more productivity supplier means producing more. Therefore, downstream firm produce more when search cost decrease. It showcase, in this restrictive model, search cost also functions as expansion cost.

On top of that, I can show that the lowest productivity  $z^R$  where this downstream firm still remain active can be calculated by solving:

$$\frac{\kappa}{1 - F[z_u^R(z^R)]} = \int_{z_u^R(z^R)}^{\bar{z}_u} [\pi(z_u, z^R) - T(z_u, z^R)] dF(z_u)$$

And keeping everything but search cost  $\kappa$  constant,  $z^R$  increase with  $\kappa$ :



It shows that when search cost decrease, less productive firm can also enter the intermediate good market and produce. This showcase search cost can act as entry barrier.

## 5.2 Full Partial Equilibrium Model Simulation

**TODO:** Add GE

The full model have no closed form solutions, so I resolve to numerical method. After VFI, one important step is to do simulation.

I simulate the full model with various different parameters to understand 1) how does search cost change allocative efficiency and concentration 2) how does economy in different condition subject to this friction.

I first initiate a set of downstream firms with productivity  $z_d$  according to the distribution  $F(z)$ . Given a set of parameters and the policies, I simulate forward. The downstream firms choose to search according to the search policy and its state. The downstream firms will get a random draw of supplier  $z_u$  and gain intermediate good and pay according to the policy functions that I solved previously. I then obtained a panel of simulated inputs  $xq$ , payments to suppliers  $T$  and search cost  $\kappa$ .

I put limit on extrapolation. Therefore, firms with very high productivity will be dropped out from the simulation. It is on one hand, problematic, as I want to study concentration where super large firms are important. However, if it is unclear if the extrapolation will be very correct, if it is very far from the policy grids.

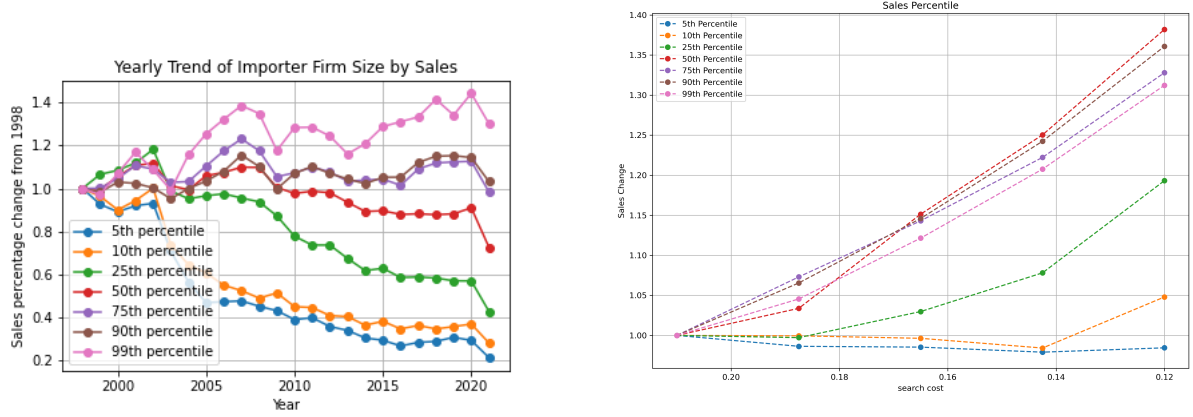
I haven't done the calibration and I use the following parameters:

The parameters are not calibrated, so the magnitude is off, but the model gives correct prediction on the direction of relationships between variables. More is in the appendix.

Variable	Values
Buyer's Bargaining Power $\theta$	.5
Convexity of search cost $\lambda$	1.05
Final Market Elasticity $\epsilon$	5
Intermediate Elasticity $\rho$	.8
Intermediate Share $\alpha$	.39
Distribution of $z$	$\log N(0,1)$
Distribution of $z_u$	$\log N(0,1)$
search cost $\kappa(0)$	[.03, .21]

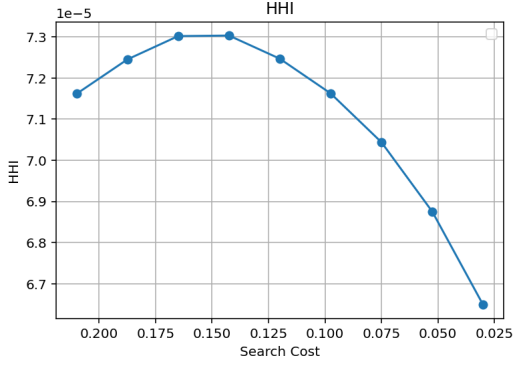
Table 1:

### 5.3 Market Concentration



In the data, firm size distribution is increasing, that assembles the first part of the graph. On one hand, smaller firms get to enter the market, but the firms that expand the fastest are the most productive firms. However, under this parameterization, very quickly the all potential downstream entrants are active, this allows me to focus on the expansion channel. In the model, when the search cost further decrease, the growth rate of median firms catch up with the most productive firms. And then eventually, the growth rate of even smaller firms catch up. The reason is the discrete increase of suppliers and the concavity of profit function. When search cost is high, let's assume the most productive firm have 20 suppliers while the median firm has 1. While adding one extra supplier is not a lot for a firm with 20 suppliers, a increase from 1 to 2 is a big jump. That's why at the start big firms expand the quickest. However, when the search cost decrease enough that less productive firms also start to expand, they expand quicker because the big firms have less incentive to further grow because of the concavity of the profit function.

As a result, the market concentration, measured by HHI, is non-monotonic with search cost, as shown in the following graph.



## 5.4 Welfare Implication of Market Concentration

I also attempt to answer the question rather this market concentration is socially efficient.

I gain intuition from the PE model, in which I continue to assume no quality difference across suppliers I perform an counterfactual exercise where a social planner can freely reallocate intermediate good  $X$  after all search and bargaining is done. The planner maximize the aggregate output, given the fixed amount of intermediate good contracted  $\bar{X}$  and the fixed labor supply  $\bar{L}$ , by choosing input and labor that each firm  $d$  uses:

$$\begin{aligned} \max_{X_d, l_d} & \left( \int (z_d (Q_d X_d)^\alpha l_d^{1-\alpha})^\rho dF(z_d) \right)^{\frac{1}{\rho}} \\ \text{s.t.} & \int l_d dF(z_d) = \bar{L} \\ & \int X_d dF(z_d) = \bar{X} \end{aligned}$$

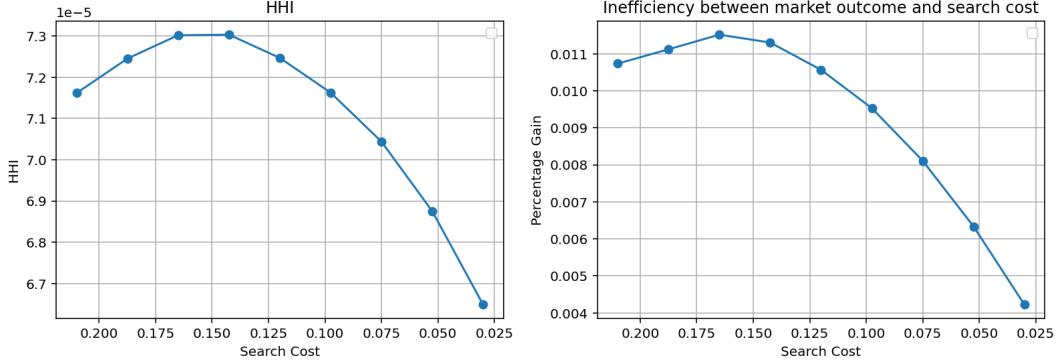
Solving for the optimal conditions, there are closed form solutions for the planner's allocation and final output:

$$\begin{aligned} \implies X_m &= [\bar{X} \left( \frac{z_d l_m^{(1-\alpha)}}{Y} \right)^\rho]^{\frac{1}{1-\alpha\rho}} \\ l_m &= \left[ \frac{\bar{L}^{1-\alpha\rho} \bar{X}^{\alpha\rho}}{Y^\rho} z_m^\rho \right]^{\frac{1}{1-\rho}} \\ Y^* &= \bar{X}^\alpha \bar{L}^{(1-\alpha)} \left\{ \int z_d^{\frac{\rho}{1-\rho}} dF(z_d) \right\}^{\frac{1-\rho}{\rho}} \end{aligned}$$

In this solution, given distribution of potential entrant  $z_d$ , the optimal allocation is market size-invariant. It provides a basis for me to compare welfare across economies



with different  $\kappa$ , even though  $\bar{X}$  and  $\bar{L}$  decrease with  $\kappa$ . I define the percentage difference between market outcome  $Y$  and planner's  $Y^*$  as inefficiency (loss from misallocation). When  $\kappa$  decrease:



It is clear that the inefficiency follows exactly the pattern of market concentration. The market outcome is more inefficient when HHI is high. As discussed before, HHI is non-monotonic with search cost, so inefficiency is also non-monotonic with search cost.

The model match the data in the starting part of the graphs. It suggests that in reality, the benefits of decreasing search cost has been mainly captured by the most productive firms. This increases market concentration and the aggregate output. However, the allocation of production factor has become less efficient. It gives interesting policy suggestions. That indicate trade policies has become more important with the current search technology. The results also indicate size-dependent industrial policies can be welfare improving by reducing inefficiency from misallocation.

## 6 Conclusions

I've gathered empirical facts on the intermediate-goods market from 1998 to 2021. Over this period, several patterns emerge: firm size in import market have become increasingly dispersed; input prices have grown more variable; and larger firms now import a broader variety of inputs. Furthermore, the most productive firms consistently secure lower input prices than their less productive peers. Together, these facts shed new light on the mechanisms of input price determination and its linkage to firm productivity, particularly highlighting how falling search costs within supply chains appear to drive these patterns.

In the theoretical part, I find that decreasing search cost have non-monotonic effects on market concentration and thus the allocation efficiency, despite increasing aggregate output. This provides interesting trade or general industrial policy implications. This echoes findings in recent trade literature, such as Bai et al. (2024), Berthou et al. (2019) or Bagwell and Lee (2020), that greater trade liberalization does not

necessarily lead to welfare gains in a frictional economy. These insights underscore the necessity of designing trade policies with careful attention to their distributional consequences.

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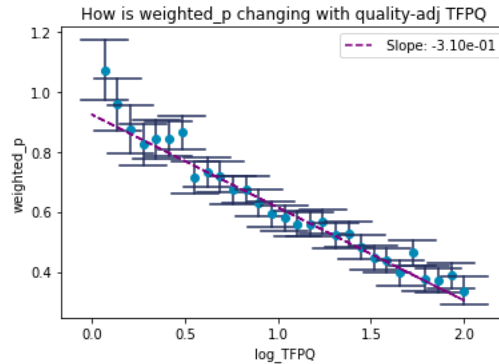
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# Data Appendix

## Non-quality adjusted TFPQ

I can directly measure output price using PPI and export data. Using this output price, I can also calculate TFPQ, but not quality-adjusted:



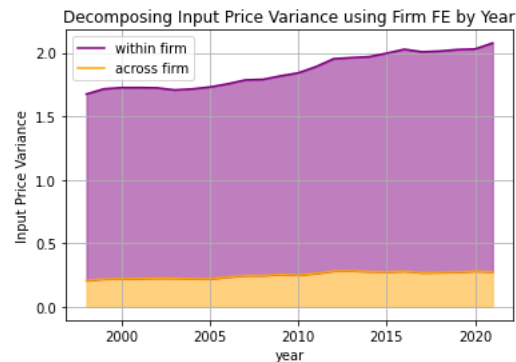
As predicted, the coefficient is also negative with a larger magnitude than the quality adjusted version. As explained in Kugler and Verhoogen (2012), firms with higher productivity usually use higher quality inputs, which dampens the quality-adjusted coefficient

## Within-Across Firm Decomposition

Also, note that it is not true that productive firm get a lower price consistently across all products they purchase. To illustrate that, I run a regression on price against only the firm fixed effect:

$$p_{fpcy} = 1 + FE_f$$

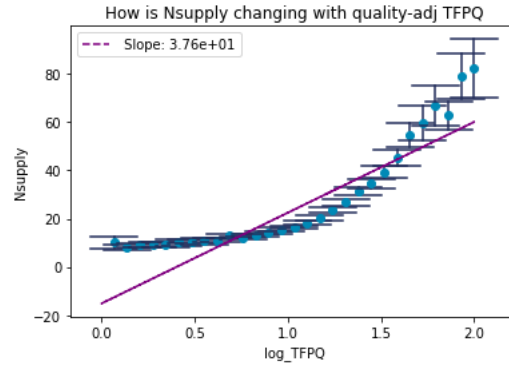
And then I decompose the variance. That part of the variance that the Fixed Effects explain is the across-firm effect and the rest is the within-firm effect. The result is rather surprising.



At odds with many model, that features firm level heterogeneity of markup or mark-down, the across-firm effect is only around 15% of the total variance. It means that a same firm can purchase one input very cheaply, but another expensively.

## TFPQ and Variety

Another productivity-related empirical fact is that more productive firms purchase more variety. It is another piece of evidence that it is likely that productive firms also benefit from the decreasing search cost because they search more. When the productive firms expect to produce more, they also have more incentive to search for the most efficient suppliers and also have multiple of them. Whereelse, the less productive suppliers even they finally gain access to the market,



## Realized Price and Quantity Relationship

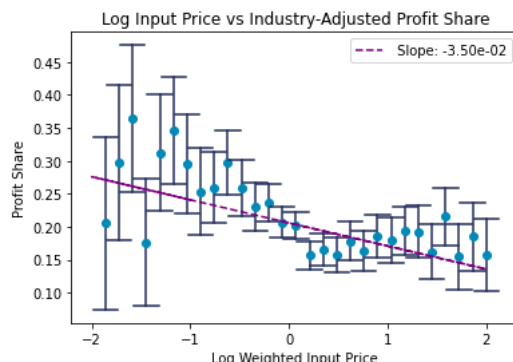
Another distinct feature of the intermediate good market is the negative correlation between price and quantity. It can be a result by many causes, such as non-linear pricing, market power or simply downward sloping demand curve as it is yearly data. However, it is a complement evidence to my claim in last subsection. More productive firms will in expectation buy more, therefore find more efficient suppliers and pay lower unit price.



A major concern is measurement error of quantity can mechanically create correlation between price and quantity (Deaton (1988)), as I use value and quantity to back out unit price. In the dataset I use, around 25% of observations have both variable. I assume that it is unlikely that both quantity measures have measurement error at the same time. Under this assumption, I compute unit price with one measure and regress it on the other quantity as a robustness check against such potential error. In this exercise, the shape and coefficient slightly changes, but the most important negative correlation is robust against measurement error.

## Input Price and Economic Outcome

Input price also directly affect the firm's profitability directly. I find a negative correlation between firm-level input price and profit share, even controlling for industry Fixed Effects. It means that the standard CES model with constant markup probably misses important features of the intermediate good market.



## Quality Differences

As researchers such as Kugler and Verhoogen (2012) show that it is an important channel of input price differences, I estimate the dispersion of input price that is generated from quality differences.

I use a reduce form approach that utilizes the Rauch (1999) classification, where he separates goods into 3 classes: traded in organized exchange, traded with referenced price and differentiated goods. Organized exchange goods also included products that have an organized exchange but can also be traded decentralized. Some examples are banana, wool or other basic agriculture products. Those products, however, have very little quality differences and therefore are suitable reference goods to estimate quality differentiation.

To link the Rauch's classifications that is based on SITC Rev. 2 the main dataset which uses CN code, I refer to the HS-SITC conversion tables provided by UNSD<sup>2</sup>.

<sup>2</sup><https://unstats.un.org/unsd/classifications/Econ>

With resale	Std Dev
Organized Exchange	.9
Referenced Price	1.18
Differentiated Goods	1.44

The CN code shares the first 6 digit with the HS code system and I assume all products that shares the first 6 digits should get the same Rauch classifications. I look at the price standard deviation of the 3 groups of products:

We can see that while differentiated goods have higher standard deviation, some part of the dispersion persist also in the “organized exchange” category. This exercise shows that quality difference cannot explain all the differences in input prices.

The important thing is there is no trend of quality differences year by year? As non-differentiated goods have minimal quality variation, I can define the quality gap as the variance gap between differentiated and non-differentiated goods. Within my observed years, there are no big changes of quality gap. By weighted average, there are no change at all. Also, the composition of differentiated goods as a fraction of all imported goods is stable at 74% ~ 76%. One way or the other, it is not sufficient to explain the price variance increase.

## Background Volatility

To check the how much of the price differences are unrelated to buyer characteristic, I look at currency volatility as one representative background noise. I construct a currency-year volatility index (against Swedish Krona) using data from the Swedish Riksbank. I then linked the currency volatility to the volatility.

I then run a regression against the variance of price:

$$\text{Var}(\log(\frac{P_{pc}}{P_p})) \sim \text{volatility}_{cy}$$

I find very little relationship between the 2 variables. (Need check)

## Theory Appendix

### Structural Estimation - Quality

Before I get to the counterfactual, note that even though quality-adjusted inputs  $XQ$  have been treated as one variables through out the model solving stage. However, they have different implications when I need to calculate welfare, misallocation etc.

The reason is quality  $Q$  goes into the consumer's welfare with a power  $\alpha$ . Decomposing  $X$  and  $Q$  also help me to understand the real price dispersion.

However, input quality is in general unobservable in the data, but I have backed out output quality in the empirical section regarding productivity. Assuming the model is correct and quality distribution is constant across suppliers of all efficiency  $z_u$ , there is a mapping between input and output quality. This has been documented in extensive literature such as XXX and XXX. From the data, I get calculate the moments  $(\mu_Q, \sigma_Q)$  of the downstream quality distribution. One caveat is that I mostly observe output quality of bigger firms. I make the log normal assumptions and calculate by XXX.

I then try to solve for the moments  $(\mu_q, \sigma_q)$  of the upstream quality distribution, which I assume to be log normal. I guess and moments, then simulate a panel of quality  $q_u$ . As I have already simulated the corresponding  $XQ$  and  $xq$  in the previous section, I can solve for the simulated downstream  $Q$  by the following equations:

$$\begin{aligned} q &= \frac{xq}{x} \\ XQ &= \left( \sum_{u \in U_i} x_u^\rho q_u^\rho \right)^{\frac{1}{\rho}} \\ X &= \left( \sum_{u \in U_i} x_u^\rho \right)^{\frac{1}{\rho}} \\ Q_i &= \frac{1}{X} \left( \sum_{u \in U_i} x_u^\rho q_u^\rho \right)^{\frac{1}{\rho}} \end{aligned}$$

I then estimate the simulated moments of  $Q$ , compare it to the empirical moments and update the upstream moments  $(\mu_q, \sigma_q)$  until the simulated and empirical moments are close enough.

## Closed Form Solution for Important Variables

Output Price and output:



$$\begin{aligned}
& \max_{p,y} py - w \left( \frac{y}{zX^\alpha} \right)^{\frac{1}{1-\alpha}} \\
& \text{s.t. } yQ^{\alpha(1-\epsilon)} = C \left( \frac{p}{P} \right)^{-\epsilon} \\
\implies & \max_p p^{1-\epsilon} \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} - wp^{\frac{\epsilon}{\alpha-1}} \left( \frac{\frac{C}{P^{-\epsilon}}}{Q^{\alpha(1-\epsilon)} zX^\alpha} \right)^{\frac{1}{1-\alpha}} \\
& D_p : (1-\epsilon) \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} p^{-\epsilon} = \frac{w\epsilon}{\alpha-1} p^{\frac{\epsilon-\alpha+1}{\alpha-1}} \left( \frac{\frac{C}{P^{-\epsilon}}}{Q^{\alpha(1-\epsilon)} zX^\alpha} \right)^{\frac{1}{1-\alpha}} \\
& p^{\frac{1+\alpha(\epsilon-1)}{1-\alpha}} = \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{\alpha}{1-\alpha}} \left( \frac{1}{Q^{\alpha^2(1-\epsilon)} zX^\alpha} \right)^{\frac{1}{1-\alpha}} \\
& p = \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{\alpha}{1+\alpha(\epsilon-1)}} \left( \frac{Q^{\alpha^2(\epsilon-1)}}{zX^\alpha} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} \\
& y = \frac{C}{Q^{\alpha(1-\epsilon)}} \left( \frac{p}{P} \right)^{-\epsilon} \\
& = \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon(1-\alpha)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{-\epsilon\alpha}{1+\alpha(\epsilon-1)}} \left( \frac{1}{Q^{\alpha^2(1-\epsilon)} zX^\alpha} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} \\
& = \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon(1-\alpha)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} \right)^{\frac{1-\alpha}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon}{1+\alpha(\epsilon-1)}}
\end{aligned}$$

Profit Function:

$$\begin{aligned}
\pi(z, QX) &= py - w \left( \frac{y}{zX^\alpha} \right)^{\frac{1}{1-\alpha}} \\
&= \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} p^{1-\epsilon} - w \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} \left( \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1-\alpha(\epsilon-1)}{1+\alpha(\epsilon-1)} \cdot \frac{1}{1-\alpha}} \\
&= \left\{ \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} - w \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} \right\} \left( \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \\
&= \left[ \frac{1+\alpha(\epsilon-1)}{\epsilon} \right] \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \\
&= \left[ \frac{1+\alpha(\epsilon-1)}{\epsilon} \right] \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{(1-\alpha)(1-\epsilon)}{1+\alpha(\epsilon-1)}} \left( \frac{C}{P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} (zX^\alpha Q^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}}
\end{aligned}$$

The labor cost is:

$$\begin{aligned}
w \left( \frac{y}{zX^\alpha} \right)^{\frac{1}{1-\alpha}} &= \left( \frac{C}{Q^{\alpha(1-\epsilon)} P^{-\epsilon}} \right)^{\frac{1}{1+\alpha(\epsilon-1)}} w \left( \frac{w\epsilon}{(\epsilon-1)(1-\alpha)} \right)^{\frac{-\epsilon}{1+\alpha(\epsilon-1)}} (zX^\alpha)^{\frac{\epsilon-1}{1+\alpha(\epsilon-1)}} \\
&= \frac{(\epsilon-1)(1-\alpha)}{\epsilon} py \text{ or } \left[ \frac{(\epsilon-1)(1-\alpha)}{1+\alpha(\epsilon-1)} \right] \pi
\end{aligned}$$

## Concavity of Bargaining Problem

Given all possible  $x$ , I can use FOC to find  $T$  because the following function is strictly concave (as long as it is defined):

$$\begin{aligned}
 B(\delta T_1 + (1 - \delta)T_2) &= (V(x) - \delta T_1 - (1 - \delta)T_2)^\theta (\delta T_1 + (1 - \delta)T_2 - K(x))^{1-\theta} \\
 \exp &\sim \theta \log((\delta + 1 - \delta)V(x) - \delta T_1 - (1 - \delta)T_2) \\
 &\quad + (1 - \theta) \log(\delta T_1 + (1 - \delta)T_2 - (\delta + 1 - \delta)K(x)) \\
 \exp &> \theta [\log(\delta(V(x) - T_1)) + \log((1 - \delta)(V(x) - T_2))] \\
 &\quad + (1 - \theta) [\log(\delta(T_1 - K(x))) + \log((1 - \delta)(T_2 - K(x)))] \\
 &= [\log(\delta^\theta (V(x) - T_1)^\theta \delta^{(1-\theta)} (T_1 - K(x))^{(1-\theta)})] \\
 &\quad + [\log((1 - \delta)^\theta (V(x) - T_2)^\theta (1 - \delta)^{(1-\theta)} (T_2 - K(x))^{(1-\theta)})] \\
 &= e^{\log(\delta B(T_1))} e^{\log((1-\delta)B(T_2))} \\
 &= \delta B(T_1) + (1 - \delta)B(T_2)
 \end{aligned}$$

Because log is a strictly concave function.. As long as  $V(x) \geq K(x)$ , the solution exist and is unique. I can get a function  $Bargain(x)$ , which is the best value given  $x$  and then I just find the arg max. Proof: Call  $T(x^o) = \arg \max_T B(x^o, T)$ , which is proved to be unique. Now assume  $(x^*, T^*)$  is the maximizer of function  $B$  and  $B(x^*, T^*) \geq B(x', T')$ . Let's say  $T^* \neq T(x^*)$ , then  $B(x^*, T^*) \geq B(x^*, T(x^*)) \implies T(x^*)$  is not the unique  $\arg \max_T B(x^*, T)$ , which cannot be true. Thus contradiction.

## Static Moments Matching

Here is the data moments matching for PE model:

